

Fig. 1 Example of nonuniqueness.

One is tempted to speculate that, since the numerical instabilities only grow for increasing y , it is permissible to integrate the system [Kao's Eqs. (10-14)] inward from the shock to the body. On closer inspection, however, it becomes clear that one is merely trading existence difficulties at one boundary for uniqueness trouble at the other. That is, if arbitrarily small errors in conditions near the body grow to very large errors at the shock, then there are certain large changes that may be made at the shock which will have no effect at the body. Indeed, reference to the preceding stability equation (1) shows that, as $V_1 \rightarrow 0$, V_1' decays more and more rapidly toward the limit $2U_1$. Since $V_{1b} = U_{1b} = 0$ are prescribed as boundary conditions in both Refs. 1 and 2, one concludes that $V_{1b}' = 0$ as the solution of the equations. Nevertheless, both papers treated $V_{1b}' = 0$ as an independent boundary condition. In the absence of further conditions, the resulting solutions are not unique.

The lack of uniqueness is illustrated in accompanying Fig. 1, which shows a solution from Ref. 2 at $Re = 10$, together with an alternate solution of the same equations and boundary conditions. A quite similar solution to Ref. 2 is given in Ref. 1 (e.g., the standoff distance is identical). The steep pressure drop immediately behind the shock is a particularly suspicious behavior, since it indicates greater viscous stresses there than elsewhere. Such a pressure drop is not present in the case of a related treatment of the viscous blunt body problem in Ref. 4, where the solutions are made unique by the introduction of an additional boundary condition consisting of mass balance at the shock.

References

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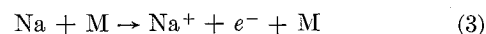
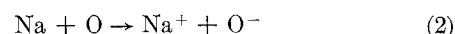
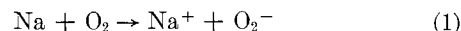
Comments on "Nonequilibrium Sodium Ionization in Laminar Boundary Layers"

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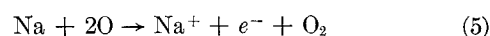
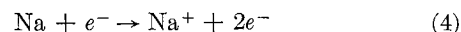
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TWO comments on Kane's¹ treatment of sodium ionization in boundary layers are warranted; his chemical system needs revision, and his method of calculation can be greatly simplified by an additional assumption.

In Lenard's work,^{2,3} Bortner's⁴ updated chemical system was used; the principal reactions turned out to be two-body reactions:



(The electrons detach almost instantly from the preceding negative ions; the "retarded" diffusion of these ions³ had no significant effect.) All but one of the preceding reactions differ from the three that were considered by Kane,¹ which were, in addition to Eq. (3),



The dominant reaction in Ref. 1 was Eq. (5), which is a three-body reaction. In Ref. 3, where a 10-reaction system was used, and reactions (3-5) had rates substantially different from those used by Kane,¹ reaction (2) was found dominant for conditions when there was sufficient atomic oxygen, otherwise reaction (1) dominated, with reaction (3) assuming a nonnegligible role.

In Lenard's calculations,^{2,3} the numerical integrations of Ref. 1 are not necessary in the treatment of the injection [Eq. (2) of Ref. 2 replacing Eq. (1) of Ref. 1] and the trace contaminant ionization [Eqs. (7) and (8) of Ref. 2 replacing Eqs. (4) and (5) of Ref. 1]. This simpler method is based on the additional assumptions of "similar profiles" for all reaction products and on the fact that the peaks of these profiles coincide with the peak temperature in the boundary layer. Using this method, only the peak concentrations of the various ions and other trace reaction products need to be calculated; their profiles throughout the boundary layer can then be reconstructed by the "similarity" assumption. The validity of these assumptions has been confirmed by examining a large number of results of exact nonsimilar calculations for pure air⁵ and one representative case for the sodium-air system.³ The factors that strengthen the validity of these simplifying assumptions are a catalytic wall, a considerable absence of equilibrium, sharply peaked temperature profiles, and temperature-sensitive rates for the principal productive reactions (true for all five sodium ionizing reactions considered in Ref. 3). Using this simpler method, the treatment of nonequilibrium ionization of sodium in laminar boundary layers is described by the expression

$$\frac{C_{\text{Na}^+}^{\text{max}}}{C_{\text{Na}}^{\text{max}}(\eta_{\text{max}})} = \left[1.0 + \frac{10I_{\text{Na}}}{Sc\rho \times \left(\frac{k_1 C_{\text{O}_2}}{32} + \frac{k_2 C_{\text{O}}}{16} + \frac{k_3}{m} \right)} \right]^{-1.0} \quad (6)$$

where C_{Na^+} and C_{Na} refer to concentrations of sodium ions and total sodium (including ions and atoms), respectively. The numbering of the forward rate constants k_1 , k_2 , and k_3

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refers to the reactions described previously, and

$$[C^{\text{Na}}(0) - C^{\text{Na}}(\eta_{\text{max}})]/C^{\text{Na}}(0) = f'(\text{Sc}\eta_{\text{max}}) \quad (7)$$

where 0 and η_{max} refer to the wall and the point of maximum temperature in the boundary layer.

The number 1.0, which appears as the first term of the result in Eq. (6), accounts for the depletion of the trace contaminant; this effect was justifiably not considered by Kane.¹ Depletion was not found important for sodium³ except at the lowest altitudes and then for very long bodies only; however, it must be considered if the ionizing trace contaminant is cesium. It may be observed that, if depletion is neglected, and the reactions specified by Kane in Fig. 1 of Ref. 1 are assumed, one may obtain the generalized results of that figure (which were obtained by correlating results of numerical integrations) directly from Eq. (6) herein.

References

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IT is agreed that the chemical system described by Lenard includes more significant reactions than those originally considered.¹ However, as was mentioned in Ref. 1, the type of analysis employed there can be applied to new ionization reactions as they are identified, just as it is assumed that new chemistry will not invalidate Lenard's work.

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The evaluation of integrals which Lenard's method would eliminate is not that difficult. Requiring only an integration method such as Simpson's rule, the integration can be accomplished by hand or in a very short time on a digital computer. Furthermore, it provides a complete description of the electron density profile which is needed if one is concerned with the attenuation of an rf signal by the plasma sheath. Since he computes the electron density at only one point in the boundary layer, Lenard's analysis cannot be applied to the attenuation problem.

For the calculation of boundary-layer peak electron density, Lenard's proposed expression is indeed simpler, but a word should be said about its range of validity. It is based on the assumption that the mass fraction profiles for all the reaction products all have the same shape and that their peak is coincident with the temperature peak. The separation of these peaks will depend on many parameters such as wall temperature and catalyticity, pressure, transport properties, etc. It will also depend on the temperature dependency of the reaction being studied. The suggested rate for the most significant reaction of Ref. 1 was temperature independent so that an appreciable separation of the temperature and sodium ion peaks was noted. Lenard's expression also implies that the electron concentration peak is coincident with the ion mass fraction peak. The validity of this implication of course depends on the manner in which the density varies. In the studies on which Ref. 1 was based, a separation of these two peaks was noted for the temperature-independent reaction so that all told, for the most significant reaction of Ref. 1, it was found that the electron concentration at the peak temperature location differed by as much as 50% from the peak electron concentration in the boundary layer.

The main value of Ref. 1 is that generalized quantitative results are presented. Lenard² has drawn qualitative conclusions and has presented results for specific calculations. A generalization of his results (as he indicates can be done in the preceding comment) would significantly enhance their utility.

References

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